Schroeder diffusers work at integer multiples of a design frequency, f_0. The design frequency is normally set as the lower frequency limit. However, it is more convenient to present formulations in terms of the corresponding design wavelength, λ_0. The depth d_n of the n^{th} well is determined from the sequence via the following equation:

$$d_n = \frac{s_n \lambda_0}{2N} \tag{9.3}$$

The well depths consequently vary between 0 and approximately $\lambda_0/2$. The design frequency is not the lowest frequency at which the surface produces more dispersion than a plane surface, it is just the first frequency at which even energy diffraction lobes can be achieved. It has been shown that Schroeder diffusers reflect differently from a plane hard surface an octave or two below the design frequency.[5,6]

9.3 Some limitations and other considerations

Given the above equations, it is possible to design a diffuser to a desired bandwidth. There are some subtle details in the design that must be heeded to achieve the best possible diffusion.

If the period width (Nw) is too narrow, then at the first design frequency there is only one major lobe, and so this concept of even energy lobes is rather irrelevant. The period or repeat width is often significant in determining performance, especially when the repeat width is small. This is illustrated in Figure 9.4 where the scattering from diffusers of different period widths are shown. These are both $N = 7$ QRDs with a design frequency of 500 Hz. The well widths are 3 and 9 cm, which means that the period widths are 21 and 70 cm respectively. The number of periods for each diffuser is set so that the overall widths of the devices are the same for a fair comparison. For the narrow wells and period width, shown right, the low frequency limit of diffusion is

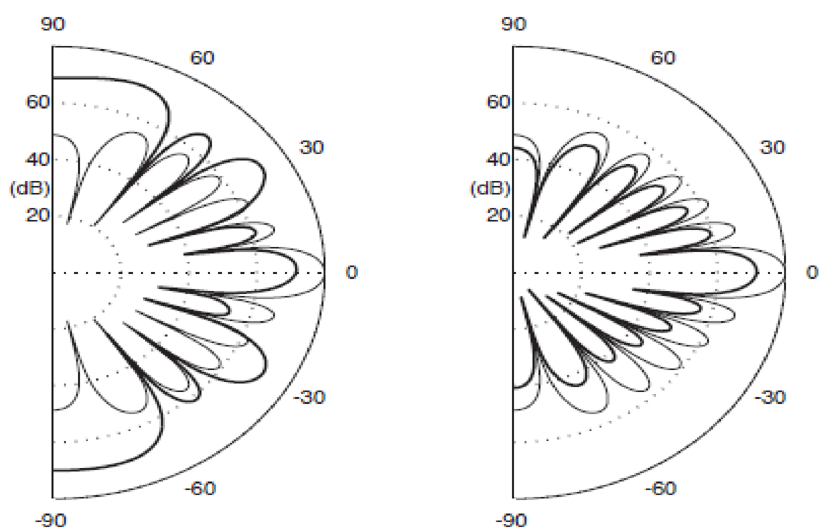

Figure 9.4 The pressure scattered from two QRDs at 1,000 Hz.
Left figure: —— QRD well width 9 cm; —— plane surface.
Right figure: —— QRD well width 3 cm; —— plane surface.
Overall width kept the same by changing number of periods.

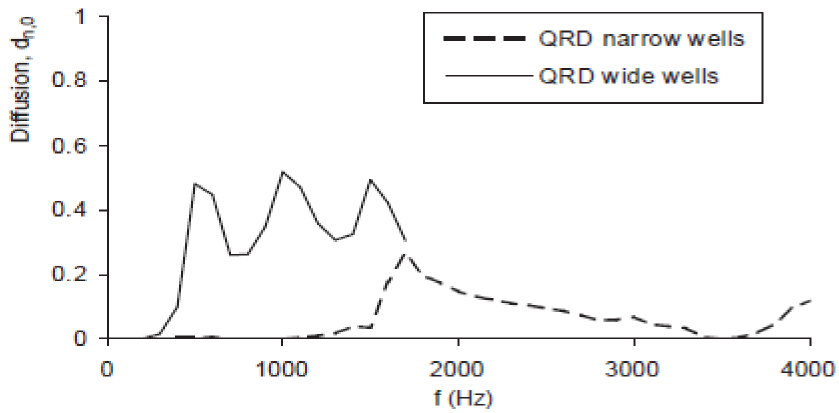

Figure 9.5 Normalized diffusion spectra for two QRDs showing that the lowest frequency at which significant diffusion occurs can be determined by period width rather than surface depth. The design frequency was 500 Hz.

determined by the period width and not by the maximum depth. This is illustrated in Figure 9.5, where the normalized diffusion coefficient versus frequency is shown. The narrow well width diffuser only starts causing significant diffusion over and above the plane surface at 1.5 kHz, which is three times the design frequency. This is roughly the frequency at which the first grating lobe appears and so is the lowest frequency where significant scattering in oblique directions is achieved. For the wide well width, the first grating lobe appears below the design frequency and so significant diffusion is created at 500 Hz and above.

For the diffuser to behave 'optimally', the device must be periodic. The lobes are generated by the periodicity of the surface. Without periodicity, all that the design equations portray is the fact that in certain directions the scattering will have a similar level. This is illustrated in Figure 9.6 where the scattering from one and multiple periods of a diffuser is compared. The directions of similar level are marked. For the periodic cases, the directions of similar level align with the lobes. For the single period case, they are just points of identical level in the polar response; the points do not align with the lobes. In this case, saying the levels are identical in some directions is

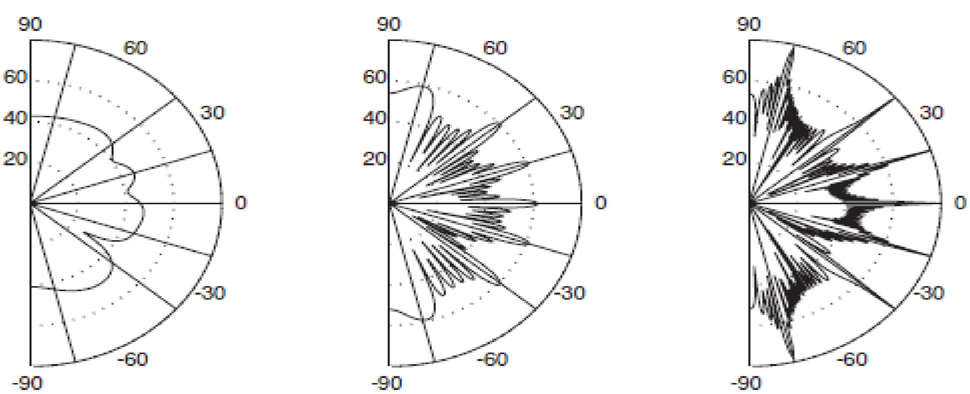

Figure 9.6 The scattering from N = 7 QRDs at 3,000 Hz for a different number of periods. Left 1 period; middle 6 periods; right 50 periods. Locations of lobes and directions of similar level marked by radial lines at ±76, ±40, ±19 and 0°.

almost a meaningless statement, because in most polar responses there will be angles where the scattering is identical to other angles. Consequently, using one period of the device spoils the point of using the quadratic residue sequence. Using one period therefore causes problems with the mathematical make-up and definition of Schroeder diffuser. However, the scattering from a single period diffuser is often more uniform than a periodic device, as Figure 9.6 shows. This issue will be returned to later when modulation is discussed.

If too many periods are included then the grating lobes become rather narrow; this leads to uneven scattering because there are large nulls present (see Figure 9.6). It must be remembered, however, that manufacturing and installation constraints are likely to mean that a narrow base shape with a large number of repeats is going to be the cheapest to build. Periodicity might also be preferred visually.

The points made in the last three paragraphs mean that the best design is one with a small number of periods, say five, to ensure periodicity, but with the diffraction lobes not too narrow. The period width must be kept large to ensure a large number of grating lobes, which then implies a reasonably large number of wells per period. Making the well width wide does not work as it can cause problems with specular-like reflections at high frequencies. Alternatively, modulation schemes can be used as discussed later in the chapter.

From the maximum frequency calculated from Equation 9.1, it might appear as though a Schroeder diffuser should have the narrowest wells possible to get the widest frequency range, but difficulty/cost of manufacture and absorption need to be considered. As the diffuser wells become more narrow the viscous boundary layer becomes significant compared to the well width and the absorption increases (see Section 9.8). Consequently, practical well widths are at least 2.5 cm, and usually around 5 cm.

The choice of prime number is limited by manufacturing cost, low frequency performance and critical frequencies. For a given maximum depth d_{max}, the design frequency achieved is:

$$f_0 = \frac{s_{max}}{N}\frac{c}{2d_{max}} \tag{9.4}$$

where s_{max} is the largest number in the quadratic residue sequence. The ratio of the largest sequence number to the prime number determines the low frequency efficiency of the device.[7] To take two examples: $N = 7$, $s_{max}/N = 4/7$; $N = 13$, $s_{max}/N = 12/13$. Consequently, an $N = 7$ diffuser will have a design frequency nearly an octave below that of an $N = 13$ diffuser. It is possible, however, to manipulate some sequences and increase the bass response. A constant phase shift can be introduced to yield a better bass response:

$$s_n = \left(n^2 + \mathrm{m}\right) \text{modulo } N \tag{9.5}$$

where m is an integer constant. Consider two $N = 13$ diffusers:

$m=0$, $s_n = \{0, 1, 4, 9, 3, 12, 10, 10, 12, 3, 9, 4, 1\}$, $s_{max}/N = 12/13$.

$m=4$, $s_n = \{4, 5, 8, 0, 7, 3, 1, 1, 3, 7, 0, 8, 5\}$, $s_{max}/N = 8/13$.

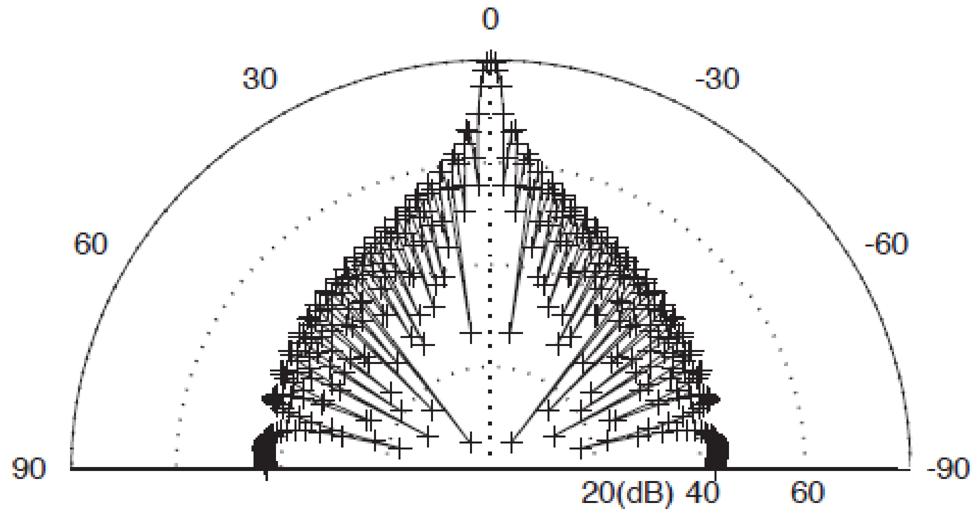

Figure 9.7 The scattering from a QRD at a critical frequency compared to a plane surface:
——— plane surface;
+ QRD.
The two lines overlay each other.

Consequently, the design frequency has been lowered by two-thirds by this simple manipulation. It must be remembered, however, that this increased performance may not be realized if the repeat width is too narrow.

For a quadratic residue diffuser, critical frequencies occur at mNf_0 where $m = 1, 2, 3 \ldots$. These are frequencies where the diffuser behaves like a plane surface because all the wells re-radiate in phase. This occurs when all the depths are integer multiples of half a wavelength. Figure 9.5 illustrates such a critical frequency happening at 3.5 kHz in the diffusion spectrum for the narrow diffuser. Figure 9.7 shows the scattering at this frequency. To avoid these critical frequencies, it is necessary to place the first critical frequency above the maximum frequency of the device defined by Equation 9.1, i.e.:

$$N > \frac{c}{2wf_0} \tag{9.6}$$